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TECHNICAL NOTES

High Peclet number heat transfer from a droplet suspended in an electric field: interior problem

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INTRODUCTION

IN 1966 G. I. Taylor [1] investigated the flow field near a dielectric droplet suspended in a second dielectric fluid exposed to a uniform electric field. He demonstrated both theoretically and experimentally that the droplet will have an interior flow pattern as illustrated in Fig. 1.

There are several modes of resistance to heat transfer from droplets, and Clift *et al.* [2] present a well written overview of the various modes governing the heat and mass transfer from droplets. The special case investigated in the present work is when the bulk of the resistance to transfer is in the droplet itself: the interior problem. More specifically, this work will investigate high Peclet number heat transfer from such droplets.

In the past few years, two works appeared which numerically investigated transfer rates for the interior of a droplet suspended in an electric field: Oliver *et al.* [3] and Manohar and Iyengar [4]. These works employed ADI (alternating direction implicit) methods to integrate the transport equation. Oliver *et al.* predicted that as the Peclet number increased, the Nusselt number would not exceed an upper boundary of about 30. Both of these investigations limited the range of Peclet numbers to 2000 or less. Thus, it has not been demonstrated (only conjectured) that the upper bound for the Nusselt number is in fact near 30 for very high Peclet numbers.

This investigation is inspired by the classic study of Kronig and Brink [5] who investigated high Peclet number mass transfer in translating droplets. They assumed that for high Peclet numbers, the concentration contours inside the droplet would be a function of time and the stream function only. With this assumption, they predicted an upper bound for the steady-state Nusselt number of 17.9. This work attempts to investigate heat transfer from a droplet suspended in an electric field using many of the same assumptions used by Kronig and Brink in their investigation of mass transfer from a translating droplet.

ANALYSIS

As the Peclet number increases, the temperature contours are assumed to correspond to the stream function values. That is

$$\lim_{Pe\to\infty} T(r,\theta,\tau) = T(\Psi,\tau) \tag{1}$$

where the stream function Ψ is given by

$$\Psi(r,\theta) = (r^3 - r^5)\sin^2\theta\cos\theta \tag{2}$$

and τ is the dimensionless time: $\tau = t\,D/a^2$.

With this assumption, heat transfer is limited to conduction orthogonal to the stream function contours. Thus, the energy equation may be shown to be of the form

$$\beta(\Psi)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial \Psi}\left(\alpha(\Psi)\frac{\partial T}{\partial \Psi}\right) \tag{3}$$

where

$$\alpha(\Psi) = \oint_{\Psi}\left(\frac{\partial \Psi}{\partial n}\right) r\sin\theta\,\mathrm{d}\varepsilon$$

or

$$\alpha(\Psi) = \oint_{\Psi} V r^2 \sin^2\theta\,\mathrm{d}\varepsilon,$$

and

$$\beta(\Psi) = \oint_{\Psi}\frac{r\sin\theta\,\mathrm{d}\varepsilon}{\left(\dfrac{\partial\Psi}{\partial n}\right)} \quad \text{or} \quad \beta(\Psi) = \oint_{\Psi}\frac{\mathrm{d}\varepsilon}{V}.$$

The partial derivative, $\partial\Psi/\partial n$, is with respect to the direction normal to the stream function contour and in towards the vortex center. The function V is the dimensionless speed of a fluid particle and is given by

$$V = \frac{\partial \Psi}{\partial n}\frac{1}{r\sin\theta}.$$

The above integrations are taken around the closed curve given by a stream function contour Ψ and are evaluated numerically with limiting cases of

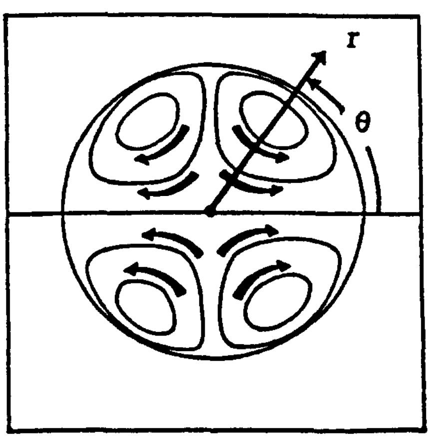

FIG. 1. Flow patterns (arrows indicate positive flow directions) for a stationary droplet in an electric field [1].

NOMENCLATURE

a	droplet radius
D	thermal diffusivity
$f(\Psi)$	see equation (8)
k	thermal conductivity
Nu	Nusselt number, $Q/2\pi akT_b$
Nu_{st}	asymptotic Nusselt number with time
Q	heat transfer rate from droplet
r	dimensionless radial coordinate
t	time
T	temperature scaled by surface temperature
T_b	bulk temperature, equation (7)
V	dimensionless speed, equation (3).

Greek symbols

$\alpha(\Psi)$, $\beta(\Psi)$	see equation (3)
θ	tangential coordinate
τ	dimensionless time, tD/a^2
Ψ	dimensionless stream function
Ψ_{max}	value of stream function at the vortex center.

$$\lim_{\Psi\to\Psi_{max}} \alpha(\Psi) = 0, \qquad \lim_{\Psi\to\Psi_{max}} \beta(\Psi) = \frac{\sqrt{(5)}\pi}{2}.$$

The boundary conditions imposed on $T(\Psi, \tau)$ are

$$T(\Psi_{max}, \tau) \text{ is finite (temperature at vortex center)} \quad (4)$$

$$T(\Psi = 0, \tau) = 0 \text{ (exterior boundary condition)} \quad (5)$$

and at

$$\Psi = 0: \frac{\partial T}{\partial \Psi} = \frac{6}{7} T_b Nu \text{ (conservation of energy).} \quad (6)$$

The quantity T_b is the bulk temperature of the droplet and may be obtained with the following relation

$$T_b = 3\int_0^{\Psi_{max}} T(\Psi, \tau)\beta(\Psi)\,d\Psi. \quad (7)$$

The maximum value for Ψ is attained at the vortex center (which is located at $r^2 = 3/5$ and $\sin^2\theta = 2/3$)

$$\Psi_{max} = \frac{4}{\sqrt{3125}}.$$

In addition to the previous assumptions, for large times the dimensionless temperature profile is assumed to behave in a self-similar manner with

$$\lim_{\tau\to\infty} T(\Psi, \tau) = f(\Psi)\exp(-\lambda\tau). \quad (8)$$

With the above assumptions, the problem reduces to finding the unknown function $f(\Psi)$ and the decay constant λ. Equation (3) thus becomes

$$\frac{d}{d\Psi}\left(\alpha(\Psi)\frac{df}{d\Psi}\right) + \lambda\beta(\Psi)f = 0. \quad (9)$$

The boundary conditions imposed on $f(\Psi)$ are

$$f(\Psi = 0) = 0 \quad (10)$$

at

$$\Psi = 0: \frac{df}{d\Psi} = \frac{12\lambda}{7}\int_0^{\Psi_{max}} f(\Psi)\beta(\Psi)\,d\Psi \quad (11)$$

and

$$f(\Psi = \Psi_{max}) = 1. \quad (12)$$

Equation (9) has been numerically solved to find the first positive value of λ for which equations (10)–(12) were true. The asymptotic Nusselt number may be found by the relation

$$Nu_{st} = \tfrac{2}{3}\lambda. \quad (13)$$

Second order finite difference methods were used to approximate equation (9), and the trapezoidal rule was used to evaluate the integrals. To check the convergence of the predicted asymptotic Nusselt number, both the size of $\Delta\Psi$ and the number of integration points (per closed stream function) used to evaluate $\alpha(\Psi)$ and $\beta(\Psi)$ were varied. The resulting predicted asymptotic Nusselt numbers are reported in Table 1.

Table 1. Convergence to Nu_{st}

$\Delta\Psi$ Grid mesh	Integration points	Nu_{st}
0.001789	40	29.925
0.000894	80	29.849
0.000447	160	29.817
0.000224	320	29.790
0.000112	640	29.789

To check the accuracy of the present procedure, it was applied to the problem of a droplet translating due to gravity. For this case the stream function is defined by

$$\Psi(r, \theta) = (r^2 - r^4)\sin^2\theta, \quad \text{with} \quad \Psi_{max} = 0.25. \quad (14)$$

Conservation of energy is imposed on the entire droplet by modifying equation (11) such that at the droplet surface ($\Psi = 0$)

$$\frac{df}{d\Psi} = \frac{3\lambda}{8}\int_0^{\Psi_{max}} f(\Psi)\beta(\Psi)\,d\Psi$$

$$\text{with limit} \quad \beta(\Psi) = \frac{\pi}{\sqrt{2}}. \quad (15)$$

With the above changes, the present model predicts an asymptotic Nusselt number of 17.903 (with $\Delta\Psi = 0.000391$ and 640 integration points). This compares with a value of 17.90 based on the work of Kronig and Brink.

RESULTS

The predicted asymptotic Nusselt number was (as expected) found to converge with an increasingly fine grid mesh and more integration points. In Table 1 are reported the predicted asymptotic Nusselt numbers as a function of the grid mesh size and the number of integration points.

Based on the previous transient work (refs. [3] and [4]), one would expect the asymptotic Nusselt number to be near 30. Based on the above analysis it may be concluded that the limit (as the Peclet number increases) for the asymptotic (steady-state) Nusselt number for a droplet suspended in an electric field is

$$\lim_{Pe\to\infty} Nu_{st} = 29.8.$$

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A close upper bound for the conduction shape factor of a uniform thickness, 2D layer

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INTRODUCTION

THE HEAT transfer by conduction through layers of uniform thickness surrounding a planar wall, a sphere, or a circular cylinder can be found from simple solutions to the one-dimensional conduction equation. Simple exact solutions are not available, however, when the uniform layer surrounds other body shapes. Yet in practice, it is often desirable to predict the heat transfer through such layers—for example, through layers of insulation applied to diverse objects, such as ducts, which come in a variety of cross-sectional shapes. Calculating natural convection heat transfer by the 'conduction layer method' (Raithby and Hollands [1]) provides yet another circumstance in which one needs to predict heat conducted through a uniform layer. The 'conduction layer method' provides its answer in terms of the thickness of a hypothetical stagnant fluid layer offering the same resistance to heat transfer as the actual boundary layer; finding the heat transfer then requires solving for the heat conducted through this layer.

Finding the heat transfer reduces to finding the layer's thermal resistance R, or equivalently the shape factor, S, where

$$S = \frac{1}{kR} = \frac{Q}{k\Delta T} \tag{1}$$

where Q is the heat transfer, ΔT is the applied temperature difference, and k is the thermal conductivity of the layer material. The present paper derives a simple approximate equation for S for cases in which the flow of heat is two-dimensional. The equation gives an upper bound for the exact value of S. The closeness of the upper bound to the exact solution and the extreme simplicity of the result make this approximate solution of practical interest.

Previous related works (Smith *et al.* [2], Balcerzak and Rayner [3], Lewis [4], Dungan [5], Laura and Susemihl [6], Laura and Sanchez Sarmiento [7, 14] and Simeza and Yovanovich [8]) all considered layers of non-uniform thickness and therefore do not strictly apply to the uniform layer problem.

To better define the problem, we first note that in the cross-sectional view (Fig. 1(a)), the layer lies between two closed curves: an inner curve C_i, and an outer curve C_o, which is spaced uniformly at perpendicular distance B from C_i. Thus curve C_o is defined as the locus of points traced by constructing outward normals to C_i and measuring out distance B along the normal. Sharp vertices in C_i that occur, for example, when C_i is a polygon (Fig. 1(b)), would appear to leave C_o undefined by this construction process, over the region near the vertex. But if we consider the vertex as a limiting case of a small circular arc near the vertex, we find that, C_o is simply filled in by an arc of radius B centered at the vertex (Fig. 1(c)).

If B is sufficiently large, and C_i is concave over parts of its length, the curve C_o may be found to be self-intersecting (Fig. 1(d)). This can happen only on concave regions of C_i; it occurs if a local radius of curvature is less than B. (Vertices having internal angles, ϕ, greater than π will always produce self-intersection of C_o, regardless of B.) If C_o self-intersects, one can question whether it is possible to actually produce a uniform layer surrounding the cylinder. Thus we exclude from the purview of the paper those combinations of B and C_i that produce self-intersection of C_o.

The present paper's derivation of an equation for S draws upon the observation of Elrod [9] that the value of S will be no greater than that derived when the shapes of the isotherms are arbitrarily assumed. The closer the assumed shapes are to those of the true isotherms, the more accurate will be the derived value of S. It happens that one particular set of assumed shapes for the isotherms satisfies many of the requirements for the true isotherm shapes and yields a very simple, general-purpose expression for the corresponding derived values of S. These particular assumed shapes are curves constructed identically to those used to construct C_o, but spaced arbitrary distance u, rather than B, from C_i, with $0 \leqslant u \leqslant B$. In this paper, the equation for S derived from this model is tested against exact results for some special cases chosen to be the most demanding on the model.

ANALYSIS

Properties of curves C_u

Let a curve constructed at constant perpendicular distance u from C_i be denoted by C_u. We first show that

$$P_u = P_i + 2\pi u \tag{2}$$

where P_i and P_u are the lengths (or perimeters) of C_i and C_u, respectively. We also show that any normal to C_i, when extended to C_u as a straight line, meets C_u at right angles. To derive these results for a smooth curve, we let $\mathbf{r}_u$ and $\mathbf{r}_i$ represent the position vectors of various points on curves C_u and C_i, respectively, as shown in Fig. 2. Then